Numerical simulation of transient vaporous and gaseous cavitation in pipelines

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Abstract *The transient vaporous and gaseous cavitation phenomena in an elastic pipeline are investigated for homogeneous liquid-gas mixture flow. It has been shown, in the case of two components having the same velocity, that modelling is also possible by considering the continuous character of the medium, i.e. without any location of column separation. The governing equations have been solved by using two finite difference schemes: the Mac Cormack's scheme and an improved new finite difference two-time step scheme. Characteristics method is used at the boundaries. The theoretical results obtained are compared and found to correlate well with similar results.*

Nomenclature

a	= wave propagation speed	*Greek symbols*	
C_0	= initial dissolved gas concentration	α	= angle of pipe inclination
C_f	= friction coefficient	β	= coefficient in relation (A6)
D_h	= hydraulic diameter of the pipe	$\in$	= average cross-sectional void fraction
e	= pipe wall thickness	Γ_g	= rate of gas release per unit volume of the fluid
E	= Young's modulus of elasticity for pipe material	ϕ	= moles number of dissolved gas/ moles number of the mixture
g	= gravitational acceleration	ρ	= density
G_r	= gas release coefficient	σ	= interfacial tension
H	= Henry's proportionality constant	γ	= constant in relation (A3)
K_r	= gas release rate coefficient	ν	= molar volume
L	= pipe length	τ	= boundary shear stress
m	= molar volume of dissolved gas		
n	= number of moles of gas per unit volume of the fluid		
p	= average cross-sectional pressure	*Subscripts*	
p_v	= vapour pressure	g	= gas
p_s	= gas saturation pressure	l	= liquid
R	= bubble radius	v	= vapour
S	= pipe cross-sectional area	a	= g + v
t	= time	s	= saturation
T	= absolute temperature	c	= cavity
V	= average cross-sectional velocity	o	= reference conditions
Wc	= volume of vapour cavity	f	= friction
x	= axial distance along the pipe	w	= wall

Introduction

The simulation of transient gas-liquid flows in pipelines has been studied for several years. This particularly important two-phase flow generally concerns nuclear, petroleum, gas industries and water distribution networks. When the pressure in a transient pipeline flow drops below a critical value, some gas bubbles can be formed. If this value is close to the saturated vapour pressure of the considered liquid, vapour cavities will appear in some particular points in the pipe, during subsequent high pressure cycles of the transient.

Column separation and gas release works, as physical phenomena, are relatively rare. Among these, Kranenburg (1974) and Wiggert and Sundquist (1979), using respectively a two-time step second order Lax-Wendroff scheme and an improved characteristics method, have considered the effect of the released gas on the flow parameters, especially on the dynamic waves. A significant limitation in each of these studies was the need to make rather arbitrary assumptions regarding the column separation modelling and the gas release rate. Baasiri and Thullis (1983) have realised an experimental quantitative study of gas release during the column separation of a homogeneous air-water transient flow. Unfortunately, the results obtained have not been compared with a theoretical model.

Privileging only the numerical aspect of the problem, Chaudry *et al.* (1990), using two explicit finite difference schemes, have resolved the conservation equation set (mass, momentum and state equations). Nevertheless, the problem was formulated without taking into account the column separation or any shocking phenomenon. However, they have showed that the effect of the dissolved gas on the parameters evolution can be predicted adequately by the homogeneous model. Recently Hadj-Tayeb *et al.* (1996), considering an explicit form of the equations, have studied the case of transient homogeneous flow in rigid and quasi-rigid pipes. The influence of the initial dissolved gas mass and the pipe wall elasticity on the pressure wave propagation showed good agreement.

So, considering the influence of dissolved gas on the pipe wall deformation, it is more suitable to take into account the gas release and the liquid and wall elasticity in the equations in order to analyse eventually the interactions between them. In the present work the calculations consider only the gas release effect.

More recently, analysing air influence on liquid transients in air pipelines equipped with valves, the experimental results of Lee (1999) have shown that when admitted air was completely expelled, a sudden rise of pressure (with a short column separation) was observed near the valve. These pressure surges were the result of the rapid valve closure. In the case of a slow valve closure the pressure amplitude has been considerably reduced.

Similarly, in a study of pressure surge due to rigidity of a pipe walls, coating and surrounding fill, Stephenson (1997) has shown that the effect of air on pipe material was a major factor in a water hammer pressures surges. This problem can generally occur during pipeline testing.

Introduction of vapour cavities in suitable locations generally yields satisfactory results. Many authors (Kranenburg, 1974; Wiggert and Sundquist, 1979), using this sample technique, have obtained a correct duration of column separation which agrees with experimental results for horizontal pipes. But when we face transient cavitation for liquid networks, the choice of cavity location for each pipe leads to an excessive computational time and a more complex program. Our formulation permits us to avoid favourably this difficulty.

Many systems of equations have been considered by several authors where the number of the unknowns generally equals the number of the equations. Analysing the two-fluid model (with four equations) and considering the oscillations frequencies of the bubbles, Kessal (1998) obtained the Wiggert and Sundquist (1979) model with an application for caviting flow. Reconsidering this work under a new general form it is proposed, here, to reduce the number of equations and calculate the three unknowns (pressure, velocity and void fraction) implicitly.

The purpose of this paper is to complete the mathematical and the physical formulation of the transient homogeneous gas-liquid flows, i.e that the effect of the wall elasticity of the pipe and the released gas and especially the column separation phenomenon, are considered by rewriting all the flow parameters in function of pressure. The later problem is, here, subject to a particular attention, by the choice of a new analytical formulation which permits us to keep the continuity of the medium. The gas release phenomenon is expressed by a coefficient calculated from the classical laws of diffusion and of Boussinesq's hypotheses (1905).

Thus, the following assumptions are made with regard to the flow regime in which gaseous cavitation is present:

- The fluid mixture is of a homogeneous, bubbly, two component nature.

- The momentum interchange between the gas and liquid is ignored. Thus for momentum considerations, the gas bubbles and liquid possess the same velocity.

- The average cross-sectional representation of void fraction, mixture velocity, and component densities can be employed.

- The mass of released gas per unit volume during the transient depends primarily on the following parameters: the pressure (p) of the liquid, the vapour pressure (p_v), the time (t), saturation pressure p_s and the temperature.

- The energy agitation is neglected.

Mathematical modelling
The mathematical model is based on a one-dimensional approach of the liquid-vapour-gas mixture flow. Generally speaking, the heat transfer process related to the cavities is faster in comparison with that of the time scale of pressure

change. Therefore, the vapour pressure and gas temperature in the cavities can be assumed to be constant. The momentum of the liquid and of gas + vapour are replaced by the momentum of the mixture.

Conservation laws
For the homogeneous model, the two components are treated as a mixture having average flow parameters. Thus mass and momentum equations of each component are:

$$\frac{\partial}{\partial t}(\rho_a \in S) + \frac{\partial}{\partial x}(\rho_a \in SV) = \Gamma_g S \tag{1}$$

$$\frac{\partial}{\partial t}(\rho_l(1-\in)S) + \frac{\partial}{\partial x}(\rho_l(1-\in)SV) = -\Gamma_g S \tag{2}$$

$$\frac{\partial}{\partial t}(\rho_a \in VS) + \frac{\partial}{\partial x}(\rho_a \in V^2 S) + \in S\frac{\partial p}{\partial x} = -\pi D_g \tau_g + g\rho_a \in S \sin\alpha \tag{3}$$

$$\frac{\partial}{\partial t}(\rho_l(1-\in)VS) + \frac{\partial}{\partial x}\left(\rho_l(1-\in)V^2 S\right) + (1-\in)S\frac{\partial p}{\partial x} = -\pi D_l \tau_l \atop + g\rho_l(1-\in)S\sin\alpha \tag{4}$$

where:

$$\rho_a = \rho_g + \rho_v$$

ρ_g, ρ_v, ρ_l and S are respectively the densities of gas, vapour or liquid and the cross-sectional area of the pipeline. Γ_g is the rate of gas production per unit volume of the mixture. It has been calculated from Wiggert and Sundquist (1979) study, as explained in the Appendix.

The void fraction $\in$ is the ratio of the volume of the gas component per unit volume of the mixture, V is the mean velocity of the mixture. The subscripts g, l and a refer to the gas and liquid and gas + vapour mixture, respectively.

The friction term for the gas and liquid phases respectively is:

$$\tau_a = \frac{1}{2}C_{f_a}\rho_a|V|V \text{ and } \tau_l = \frac{1}{2}C_{f_l}\rho_l(1-\in)|V|V$$

In this study:

$$\tau_a = \tau_g \text{ and } C_{fa} = C_{fg}$$

where C_f, p, $D_{g,l}$, $\tau_{g,l}$, and α are respectively the friction coefficient, the mean pressure, the hydraulic pipe diameter for the two fluids, the wall stress and the angle of inclination of the pipe.

Summing equations (1) and (2) and neglecting the contribution of the gas phase, the mixture momentum equation (in place of (3) and (4)) gives:

$$\frac{\partial}{\partial t}(\rho S) + \frac{\partial}{\partial x}(\rho S V) = 0 \tag{5}$$

$$\frac{\partial}{\partial t}(\rho S V) + \frac{\partial}{\partial x}(\rho S V^2 + P) = -\pi D \tau_w + \rho g S \sin\alpha, \tag{6}$$

in which ρ is the mean density of the mixture as:

$$\rho = \rho_a + \rho_l(1 - \in) \tag{7}$$

and

$$P = \int_{p_0}^{p} S dp, \tag{8}$$

where p_0 is a reference pressure.

If we introduce the usual equation of state of the wall of the pipeline as:

$$\frac{dS}{S} = \frac{D}{Ee} dp, \tag{9}$$

where e, E are respectively the pipe wall thickness and the Young's modulus of elasticity of the pipe material.

Then integration of the relation (8) with (9) yields:

$$P = \frac{S_o(p - p_o)}{1 - \frac{(p - p_o)D_o}{2eE}}. \tag{10}$$

Taking:

$$\zeta = \rho S, \tag{11}$$

equations (5) and (6) become:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x}(\zeta V) = 0 \tag{12}$$

$$\frac{\partial}{\partial t}(\zeta V) + \frac{\partial}{\partial x}(\zeta V^2 + P) = -\zeta g \sin\alpha + \zeta \pi D \tau_l. \tag{13}$$

Considering a small spherical bubble of radius R filled with vapour and surrounded by a saturated flowing liquid, and neglecting viscous terms, the equilibrium condition is:

$$p_g = p - p_v + \frac{2\sigma}{R}. \tag{14}$$

where p_g, p_v and σ are the gas and vapor pressures and the interfacial stress σ, respectively.

The equation of state of the gas and the liquid respectively is:

$$\frac{d\rho_{g,l}}{\rho_{g,l}} = \frac{1}{K_{g,l}} dp, \tag{15}$$

where $K_{g,l}$ is the bulk modulus of elasticity of each component.

Combination of relations (1) and (5) allows us to write (Kessal, 1987):

$$\frac{1}{\rho}\frac{d\rho}{dt} - \frac{1}{\rho_g}\frac{d\rho_g}{dt} - \frac{1}{\in}\frac{d\in}{dt} = -\frac{\Gamma_g}{\rho_g \in}.$$

Integration of the above equation with the aid of relations (10) and (11) allows us to express the parameter ζ in the following form:

$$\zeta = \rho_0 \frac{\in}{\in_0} P^\bullet \frac{1}{\left(\frac{1}{\sqrt{S_0}} - \frac{p-p_0}{eE\sqrt{\pi}}\right)^2} G_r, \tag{16}$$

where the coefficient G_r is function of gas release rate per unit volume of fluid Γ_g, whose time evolution may be found in the Appendix.

$$G_r = \frac{1}{1 + \int\limits_0^t \frac{\Gamma_g}{\rho\in}} dt \tag{17}$$

and

$$P^\bullet = \frac{p - p_v + \frac{2\sigma}{R}}{p_0 - p_{vo} + \frac{2\sigma}{R}}. \tag{18}$$

In this equation the radius of the bubble is determined by the equilibrium equation, in which the dynamic effects are neglected. This is justified if only transient phenomena are considered to have a larger time scale than the natural period of bubble oscillations.

Considering equations (7) and (15), the void fraction can be expressed as:

$$\in = \frac{\rho_{lo}\exp\left(K_l^{-1}(p - p_0)\right)}{\rho_{lo}\exp\left(K_l^{-1}(p - p_0)\right) - \rho_v + \left(\frac{\rho_0}{\in_0} - \rho_{go}\right)}. \tag{19}$$

Introducing a dimensionless form for ζ as: $\zeta^\bullet = \dfrac{\zeta}{\rho_o S_o}$ then equation set (12)-(13) becomes:

$$\frac{\partial \zeta^\bullet}{\partial t} + \frac{\partial}{\partial x}(\zeta^\bullet V) = 0 \tag{20}$$

$$\frac{\partial}{\partial t}(\zeta^\bullet V) + \frac{\partial}{\partial x}(\zeta^\bullet V^2 + P') = -\zeta^\bullet g \sin\theta - 2C_f \frac{\zeta^\bullet}{D_h}|V|V \tag{21}$$

where:

$$P' = \frac{p - p_o}{\rho_o S_o}. \tag{22}$$

By introducing relation (10) this equation becomes:

$$P' = \frac{p - p_o}{\rho_o}\left(1 - \frac{(p - p_o)D_o}{2eE}\right)^{-1}. \tag{23}$$

Neglecting the interfacial tension σ and introducing some simplifications, we can finally write:

$$\zeta^\bullet = \frac{P^\bullet[1 + Eo'(P^\bullet - 1)]}{\in_o' + (1 - \in_o')P^\bullet}G_r \tag{24}$$

where:

$$\in_o' = \in_o\left[1 - (Eo1 + \frac{\rho_{v_o}}{\rho_{l_o}})\right]$$

$$Eo' = Eol + \frac{D_o}{e}Eos$$

$$Eos = \frac{\rho_o - \rho_v}{E} \quad \text{and} \quad Eol = K_1^{-1}(p - p_v).$$

From the above equation the ratio $P^\bullet$ can be expressed in function of $\zeta^\bullet$, as:

$$P^\bullet = (1 - \in_o')\zeta^\bullet - (1 - Eo') + \sqrt{\frac{[(1 - Eo')\zeta^\bullet - (1 - Eo')]^2 + 4\in_o'\zeta^\bullet Eo'}{2Eo'}} \tag{25}$$

and

$$P' = \frac{(P^\bullet - 1)Pro}{1 - (P^\bullet - 1)Dro} \tag{26}$$

where:

$$\text{Pro} = \frac{\text{Eos}}{\rho_o} \quad \text{and} \quad \text{Dro} = \frac{D_o}{2e}\text{Eos.}$$

Considering the relative complexity of the above relations, the terms Eos, Eol, Eo', Pro, and Dro have been used in order to facilitate digital computing.

The method of characteristics may be used to transform the equation set (20) and (21) to an ordinary differential equation set (Courant and Hilbert, 1962). Then resulting characteristics form (or compatibility equations) is:

$$\delta P' \pm \zeta^\bullet a \delta V = \mp(\zeta^\bullet ag\sin\theta - \frac{2C_f}{D_h}\zeta^\bullet a|V|V)\delta t. \tag{27}$$

These equations are associated with the following characteristics directions, respectively:

$$\frac{dx}{dt} = V \pm a. \tag{28}$$

The differential operator is written as:

$$\frac{\delta}{\delta t} = \frac{\partial}{\partial t} + (V \pm a)\frac{\partial}{\partial x}, \tag{29}$$

where the parameter a is the wave propagation speed in the mixture. It can be expressed in the following suitable form:

$$a = \left(\frac{1}{\dfrac{d\zeta'^\bullet}{dP'}}\right)^{\frac{1}{2}}. \tag{30}$$

Using equations (20) and (21), the celerity a becomes:

$$a = \frac{A_1}{1 - \dfrac{D_o}{2e}\text{Eos}(P^\bullet - 1)}\sqrt{\frac{\dfrac{E}{\rho_o}\text{Eos}}{(A_2 + P^\bullet E_o')A_1 - A_2 P^\bullet(1 - \epsilon_o')}}. \tag{31}$$

where:

$$A_1 = \epsilon_o' + (1 - \epsilon_o')P^\bullet$$
$$A_2 = 1 + \text{Eo}'(P^\bullet - 1)$$

It can be noticed, from the above equation, that the presence of amounts of free gas in liquids leads to a very strong influence of pressure on wave speed, which drops with low pressure and regains its high amplitude as the pressure rises.

Besides, this equation retains pipe elasticity and portrays the effect of gas content and pressure on acoustic waves.

Numerical resolution

Taking into consideration the conservative form of the equation set (20)-(21), this paper considers two appropriate finite difference schemes. The first one concerns a two-time step predictor-corrector scheme for the interior mesh points: an improved Lax-Friedricks scheme as a predictor and a leapfrog scheme as a corrector. The first step is obtained by using a stabilising procedure which corresponds, in fact, to the addition of a dissipate term proportional to the second derivative (Hirch, 1997). Unfortunately, this scheme causes considerable damping of the waves, owing to its first-order accuracy. This would lead to too low values for the maximum pressures. Second order accuracy can be obtained by adding an adapted second step (leapfrog scheme). The numerical damping by this two-step scheme is appreciable, provided that a sufficient number of mesh points is chosen.

It is relatively well established that using the well known family of schemes S_β^α for arbitrary α and β (Lerat and Peret, 1973) can be recommended for their numerical resolution of the equation set (20)-(21). For values of $\alpha = 1$ and $\beta = 0$ we obtain the predictor-corrector Mac Cormack's scheme (used in this study) which is generally employed in many problems of gas dynamics. During the numerical applications of the two previous schemes the characteristics method is used for the boundary conditions.

Considering that the column separation phenomenon is not only a hydrodynamics problem but also a phase change problem, it is not necessary to use the high resolution schemes. In addition, it is now well known, as was reported by Fletcher (1997), that high-order schemes produce a more accurate solution than FCT algorithm, but a more sophisticated coding and a less economical computational time. However, FCT algorithms are more accurate than artificial viscosity. So, in order to accomplish a suitable treatment of the shock problem considered herein (abrupt valve closure), an FCT algorithm is added to the two above predictor-corrector schemes as additional steps.

Applying the above predictor-corrector schemes to equations (20) and (21) gives the values of $\zeta^\bullet$ and V. A computational procedure is employed to carry out from equations (25), (19) and (A16) unknowns p, α and G_r. The boundary conditions are calculated from the compatibility relations (27) with a spatial linear interpolation (Lister, 1960). It is important to notice that, in the case of pipeline networks it is not necessary to compute the cavity volume for each duct (or column separation). This has been avoided by introducing the concerned boundary condition as a subroutine in the main program.

Numerical computations have been facilitated by considering only the finite difference scheme in the main file (which gives $\zeta^\bullet$ and V). At the boundaries the parameters P′ and V are obtained by using the characteristics equations (27).

Applications

Wiggert and Sundquist (1979) conducted a series of tests with dissolved gas in water mixer by considering a pipe loop 295m in length and 0.0254m in diameter. Numerical results were presented for different values of dissolved gas concentration (Table I). The checked graphs concern the located points $x = 0$ and $x = 0.5*L$ of the pipeline. The transients are due to an abrupt closure of the valve at the upstream boundary of the pipeline (i.e. $X = 0$).

In our work, six experimental runs were numerically simulated on a digital computer using our physical model resolved by the two previous finite difference schemes. In this analysis the following conditions are imposed:

- the initial void fraction is very small;
- the initial velocity and pressures are calculated from steady-state flow;
- the gas behaviour is isothermal; and
- the valve closure is instantaneous.

Values of K_Γ, β and P_S in equation (A6) are calculated in the Appendix by using diffusion laws, aided by Wiggert and Sundquist (1979) gas release mass estimation.

Column separation and the caviting flow

In previous works, the following cavitation test had to be applied in the point where the column separation was supposed to be observed:

$$\text{If } p < p_v \text{ then } p = p_v.$$

Therefore, the cavity volume could be calculated. The technique, generally employed to calculate this volume W_c was based on mixture velocities (V_2, V_1) on either side of the column separation:

$$W_c^{t+\Delta t} = W_c^t + \Delta t(V_2 - V_1)$$

The calculation procedure ended when W_c is < 0.

The following technique permits us to avoid the above procedure. In the case of an abrupt valve closure at the upstream boundary, finite-difference form of

Value location	Initial velocity	Gas	Gas content[a] (C_o)	Reservoir pressure (K_N/M^2)	Fluid temperature ($^\circ$C)
Upstream	0.77	Air	0.02	172[b]	16
Upstream	0.77	CO_2	0.60	175[b]	16
Upstream	0.77	CO_2	1.15	175[b]	16

Table I.
Experimental conditions

Notes
[a] Ratio by volume at standard conditions
[b] Downstream reservoir

one of the equations (27) – corresponding to upstream condition – is used, where the pressure at $X = 0$ ($p_1^{t+\Delta t}$) is calculated for a velocity $V_1^{t+\Delta t} = 0$ (subscript $1 \rightarrow X = 0$). The obtained value of P' is negative which leads to negative value of $P^\bullet$. Now, deducing $P^\bullet$ from equation (26) yields:

$$P^\bullet = \frac{P'}{Pro + P'Dro} + 1.$$

If the obtained value of $P^\bullet$ is lower than zero then a condition as:

if $P^\bullet < 0$ then $P^\bullet = 0$ (or equal to a very small value)

can be imposed on $X = 0$ and on all points i of the mesh.

It can be noticed that, for this value equation (18) gives $P = P_v$, if σ is neglected. Thus, in the valve point ($x = 0$), the column separation phenomenon can be obtained numerically without employing the above-mentioned classical calculation cavity volume.

Figures 1, 3 and 5 show a comparison, as regards the pressure at $x = 0$, between Wiggert and Sundquist (1979) experiments and theoretical results (indicated by W&S in the figures) and the model described herein, assuming some quantities of initial dissolved gas C_o. It can be observed that, at first the cavity begins to grow and collapses after an outward and return of the wave. After this collapse, it can also be shown that a shock wave is then generated, which starts to propagate in the downstream direction.

Gas release has been calculated without assuming an initial number of bubbles in the mixture (Kranenburg, 1974), but by introducing an initial gas
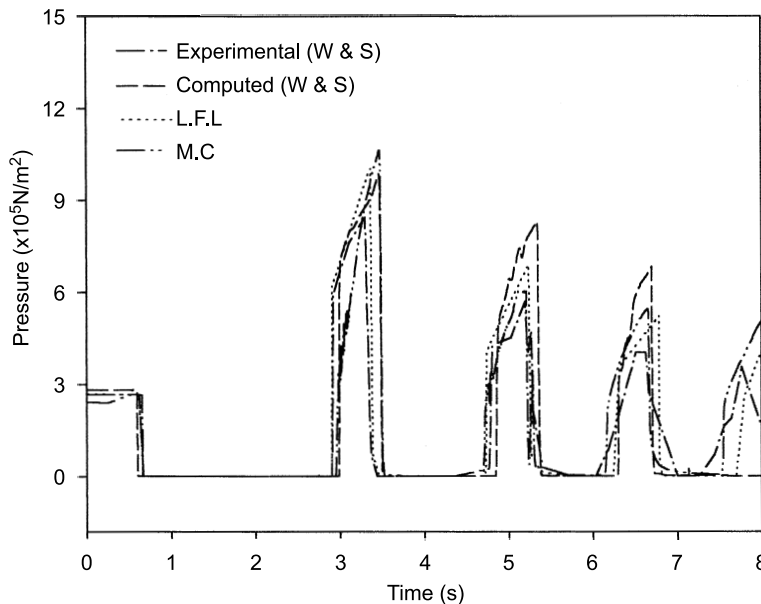

Figure 1.
Pressure response at
point $x = 0$, for $C_o = 0.02$

Figure 2.
Pressure response at
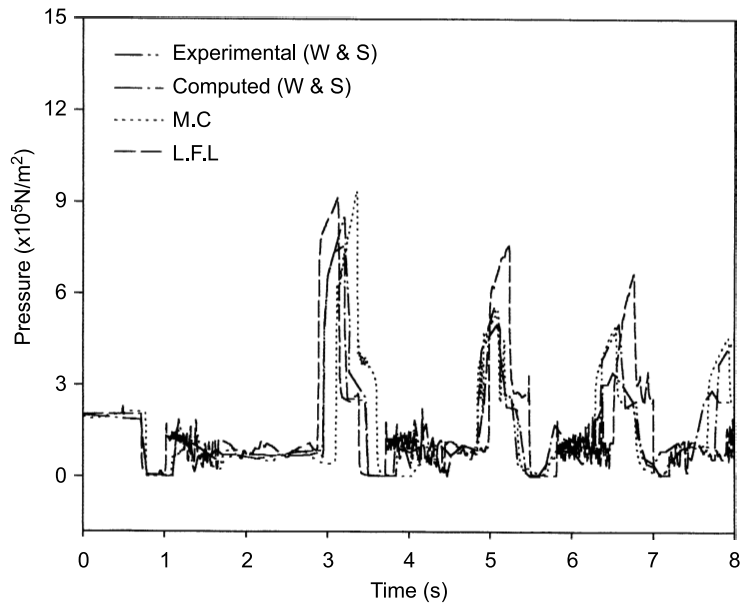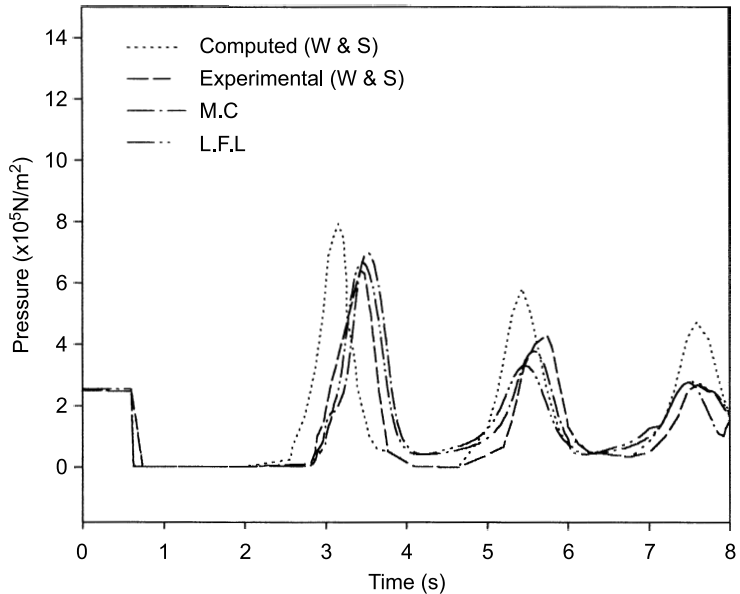point x = 0.5 × L, for
$C_o = 0.02$

Figure 3.
Pressure response at
point x = 0, for
$C_o = 0.6$

concentration C_o. The agreement is satisfactory, indicating that MacCormack's scheme (MC in the figures) and the Lax-Friedricks-leapfrog scheme (LFL scheme in the figures) described in the previous section confirm the validity of our physical model.

In these figures it seems that the gas release in the caviting flow region of the pipeline causes a considerable decrease of the duration of the subsequent
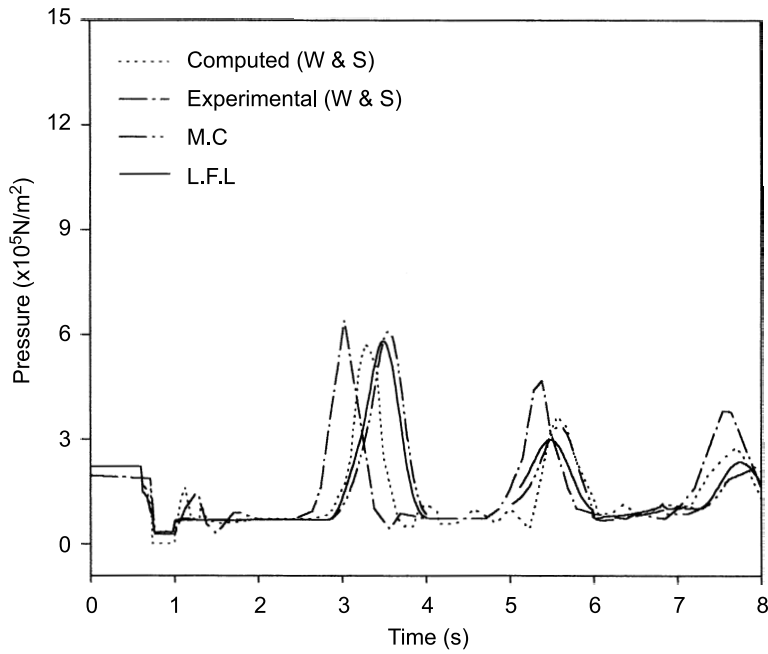
Figure 4.
Pressure response at
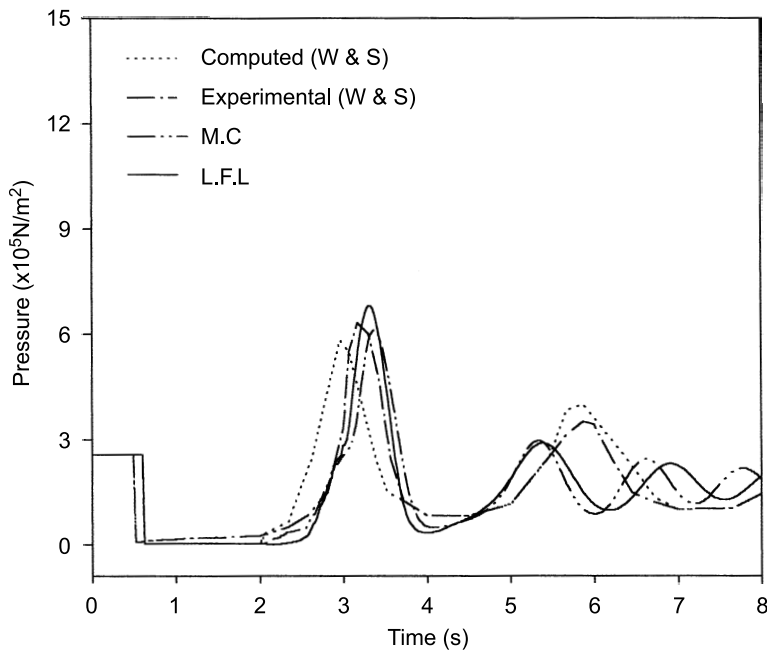point x = 0.5 × L, for
C_o = 0.6

Figure 5.
Pressure response at
point x = 0, for
C_o = 1.15

columns separation and of the pressure. This can be explained by the fact that the gas release causes the dilation of the fluid column in the caviting flow region. This region can take place freely due to the pressure drop in the adjacent cavity. Consequently, the volume of this cavity is less than in the case without gas and it decreases as C_o increases. Thus the cavity collapses at an earlier instant.

Figures 2, 4 and 6 concern the pressure responses at the midpoint of the pipeline (X = 0.5*L). It can be noticed that a small, but of short duration cavity occurs at the beginning. As in the previous figures, the increase of C_o causes appreciable damping of the successive pressure peaks since a greater initial concentration C_o leads to much gas release. Consequently, the volume of the cavity (or its duration) is smaller since the local pressure grows with gas release.

As can be observed in these figures, there is a good agreement between theoretical and experimental results for the first pressure peaks, i.e the beginning of the phenomenon. However, during the successive cavity formations the flow pattern can change. So, it can be noticed that the inclusion of gas release as a term source in equation (1) has no great influence on the flow parameters when the dissolved quantity is small; but when this quantity is larger than a certain value the dynamic parameters are so damped that the homogeneous model developed herein would require some improvements.

In the numerical plan a comparison between the two finite difference schemes used shows that MC scheme agrees better with experimental results. The small differences, observed in Figures 1-6, are certainly due to the fact that LFL scheme and Wiggert and Sundquist numerical method produce more dispersion errors.
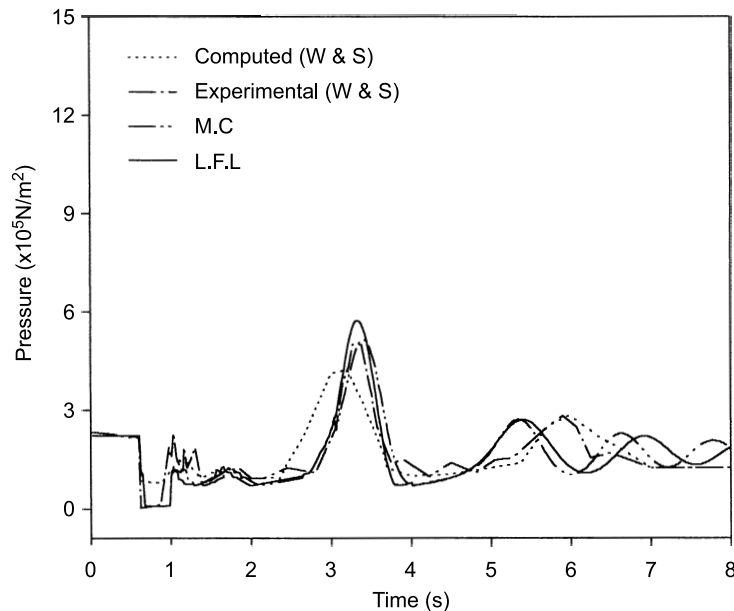

Figure 6.
Pressure response at point x = 0.5 × L, for C_o = 1.15

Conclusion

A complete homogenous two-phase flow model, with only two equations set, is performed by considering a dimensionless form of the parameters, by taking into account the elasticity of the pipe wall, the compressibility of each component of the mixture and the influence of gas release on the flow parameters. Vaporous and gaseous cavitation have been studied by resolving the obtained new equation set with two second order finite-difference schemes. Computational results, in the case of an abrupt valve closure at the upstream boundary of the pipeline, are in good agreement with those found in literature where the located cavity method is used.

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Appendix. Estimation of the gas release coefficient $G_r(t)$

Henry law for gas-liquid mixtures can be written as:

$$P_s = \phi H, \tag{A1}$$

where P_s is the gas saturation pressure of the liquid and ϕ the ratio between the moles number of dissolved gas and the moles number of the mixture. H is the Henry's proportionality constant which depends on the nature of the dissolved gas and the temperature. It can be expressed in the following simple form (Gaid, 1984):

$$H = H_{10} \times 1.02^{(T-10)}, \tag{A2}$$

where H_{10} is the Henry's proportionality constant at T = 10°C (H_{10} 5.49 × 10^4 Pa for air in water).

In his work on gas release phenomena, Kranenburg (1974) has defined a concentration C_K as the number of gas moles n per m^3 of fluid:

$$C_K = \gamma P_g, \tag{A3}$$

where γ is a constant given in moles/J.

Taking into account the relation (A1) this concentration can be written as:

$$C_K = \phi \times n. \tag{A4}$$

Wiggert and Sundquist (1979) have defined a concentration C_0 as the ratio of the dissolved gas volume per m^3 of the mixture. Thus, by considering relation (A3) this concentration, at standard conditions, takes the following form:

$$C_0 = \frac{P_s}{H} \times n \times \nu, \tag{A5}$$

where $\nu = 22.4 \times 10^{-3} \, m^3$ is the molar volume at standard conditions.

Calculation of the quantity Γ_g in the relation (17) has been outlined from the Wiggert and Sundquist (1979) study:

$$\Gamma_g = \begin{pmatrix} K_r \beta(p_s - p), p_s < p \\ 0 \qquad\quad , p_s > p \end{pmatrix}. \tag{A6}$$

In this study values of coefficient $K_\Gamma = 7.6 \times 10^{-4}$, 1.1×10^{-3} and $1.1 \times 10^{-3} \, S^{-1}$ corresponding respectively to $C_0 = 0.02$, 0.6 and 0.15, as fixed by the previous author, has been retained. Whereas, the values of β and P_s must be calculated in function of temperature.

By taking into account relation (A2), it is possible to express the coefficient β (defined by Wiggert and Sundquist (1974) in function of the molecular mass m of the dissolved gas:

$$\beta = \gamma \times m. \tag{A7}$$

Then considering relations (A4) and (A5) this coefficient becomes:

$$\beta = \frac{n \times m}{H}. \tag{A8}$$

The saturation pressure P_s can be calculated from the relation (A5).

Example of application for air dissolved in water at T = 16°C:

$$H = 6.18 \times 10^4 Pa; \quad \beta = 2.641 \times 10^{-7} \, S^2/m^2; \quad P_s = 1.01 \times 10^5 Pa.$$

Considering the relations (14) and (18) which give:

$$P^{\bullet} = \frac{\rho_g}{\rho_{g_o}},$$

so, from relation (16) we can express the gas density in the following form:

$$\rho_g \in = \zeta^{\bullet} \rho_{g_o} \in_o \frac{S}{S_o} \frac{1}{G_r}, \tag{A9}$$

where G_r is given by the relation (17):

$$G_r = \exp\left(-\int_0^t \frac{\Gamma_g}{\rho_g \in}\right). \tag{A10}$$

Time evolution of this coefficient permits to write:

$$G_r(t+\Delta t) = G_r(t) \exp\left(-\int_t^{t+\Delta t} \frac{\Gamma_g}{\rho_g \in} dt\right). \tag{A11}$$

After development this coefficient becomes:

$$G_r(t+\Delta t) = G_r(t)\left(1 - \frac{\Gamma_g}{(\rho_g \in)_t}\Delta t\right). \tag{A12}$$

Replacing the term $\rho \in$ by relation (A9), this equation becomes:

$$G_r(t+\Delta t) = G_r(t)\left(1 - \frac{\Gamma_g \Delta t}{\zeta^{\bullet} \rho_{g0} \alpha_0} \frac{S}{S_0} G_r(t)\right). \tag{A13}$$

By taking into account the relations (14) and (18) the fluid pressure becomes:

$$p = p_v + P^{\bullet}(p_o - p_{v_o}), \tag{A14}$$

replacing the ratio $\frac{S_o}{s}$, in the relation (A13), by an equation deduced from the pressure cross sectional variation (Kessal, 1987), $S = S(P^{\bullet})$:

$$\frac{S_o}{S} = \left[1 + \frac{D_o}{e}\text{Eos } (P^* - 1)\right]^{-1} \tag{A15}$$

and the quantity Γ_g from relation (A6), then the relation (A13) can take the final following form:

$$G_r(t+\Delta t) = G_r(t)\left\{1 - (K_1 - K_2 P^{\bullet})G_r(t)\frac{\left[1 - \frac{D_o}{e}\text{Eos}(P^{\bullet} - 1)\Delta t\right]}{\zeta^{\bullet}}\right\}, \tag{A16}$$

where the coefficients K_1 and K_2 are functions of p_s, p, p_o and p_{vo}:

$$K_1 = \frac{K_\Gamma \beta(p_S - p)}{\rho_{g_o}\alpha_o} \quad \text{and} \quad K_2 = \frac{K_\Gamma(p_o - p_{vo})}{\rho_{g_o}\alpha_0}.$$